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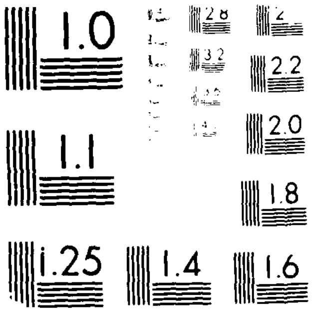
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Robust estimation technique for autoregressive and autoregressive-moving average models were developed and analyzed. Solutions were obtained on system theoretic problems associated with parameter estimation problems for time series models. An adaptive estimation techniques for a class of piecewise stationary signals was developed.

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PRINCETON UNIVERSITY
Department of Electrical Engineering
Princeton, New Jersey 08544

Final Technical Report **AFOSR-TR- 87-1793**

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Efficient Algorithms and Structures for Robust Signal Processing

Program Officer: Major Brian Woodruff, AFOSR/NM

Bradley W. Dickinson

Bradley W. Dickinson
Professor
Principal Investigator
(609)-452-4644

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Final Technical Report

The research efforts supported by AFOSR Grant AFOSR-84-0381 were directed towards development and analysis of robust estimation techniques for autoregressive (AR) and autoregressive-moving average (ARMA) models. Work on related system theoretic problems associated with parameter estimation problems for time series models and on square-root filtering for least squares state estimation applications was also carried out. Finally, an adaptive estimation technique for a class of piecewise (in time) stationary signals was developed. The motivation for our research arises from applications in signal processing including linear predictive signal modeling, signal detection, dynamic state estimation (Kalman filtering), and spectral analysis. The general goal of this research has been to put together ideas and techniques from statistics, signal processing, and system theory to bring new perspectives to such problems.

Our research on various autoregressive modeling problems resulted from a desire to relax some of the assumptions made by previous researchers, in order to broaden the domain of application of the basic technique which has proved to be useful in a range of signal processing tasks. In particular, our efforts have been directed at the goal of obtaining allowing robust estimates in the presence of outliers in the observed signal and in modeling of signals whose spectral characteristics change abruptly from time to time. These two situations provide models that have wide applications to signal processing problems encountered in practice. First, outliers represent a particularly deleterious form of noise with respect to destroying the quality of least squares estimates (as is well known in many contexts), yet models of isolated "bursty" noise effects are appropriate in nearly all signal processing applications. Second, abrupt changes in signals occur as the result of system failures (e.g. changes in dynamic response of a structure due to failure of some part) or as the result of different signal propagation paths. Failure detection and diagnosis for complex aerospace systems is a main motivation for the study of models for abruptly changing signals.

Our work has made extensive use of a particular model structure, the so-called lattice structure, for autoregressive processes. Many advantages of the lattice structure have been recognized for some time, as evidenced by the vast literature that has been built up on this topic. Nevertheless, we have made several new and important applications of lattice filters in the research supported by AFOSR. These will be explained in the course of the discussion of our various research results in what follows.

A major portion of our work dealt with the use of lattice filters in robust estimation for AR and ARMA time series models. Here a main accomplishment was to reduce the computational complexity of the iterated, reweighted least squares approach to outlier suppression; our first work on this was limited to AR models and reported in [1]. Subsequently, we were able to expand this approach to handle ARMA models as well; this has been reported in [6], and a paper describing the approach is presently under revision after review [9]. For the AR case, the lattice structure makes crucial use of partial autocorrelations to parametrize the autoregressive model, resulting in a decoupling of the least squares problems. Results in [1] show that just one or two iterations of the basic "data cleaning" procedure provides estimates with much better performance than least squares. The extension to more general ARMA models involves the use of lattice filters of length $p+q$, the sum of the AR and MA orders, together with an important discovery of the explicit relationship between ARMA parameters and partial autocorrelations [4].

The work reported in [4] was directed at a special ARMA estimation problem, namely the use of the lattice structure to obtain estimates of the AR part of an ARMA model. Such a problem arises when applying linear prediction (autoregressive modeling) for spectral estimation of sine waves in additive white noise. We showed in [4] that it is not necessary to solve a sequence of least squares problems (of order greater than or equal to the autoregressive order) to get consistent estimates; rather an extended lattice filter may be transformed to give the estimates from the solution of a single set of linear equations (of order equal to the moving average order). This result is shown to be algebraically equivalent to solution of the extended Yule-Walker equations. However, the formulation in terms of partial autocorrelations rather than in terms of the ordinary autocorrelation sequence turns out to be the crucial factor in providing good estimates.

(This was discovered during the studies which led to [9].) Thus not only does the lattice filter approach provide a computationally efficient estimation method, but it provides a much more accurate method than the "classical" extended Yule-Walker method. Of course, the principal advantage still comes when we apply the robust estimation techniques already mentioned to this more general estimation problem, since the Yule-Walker approach does not lend itself to efficient "robustification" by iterated, weighted least squares.

The final step involved in this research was the discovery that some of the intermediate quantities obtained in the fitting of the $(p+q)$—order lattice filter could be used to obtained estimates of the MA parameters directly. While these estimates are clearly biased, the fact that they are obtained without much additional computational cost makes our approach attractive for estimation of general ARMA models in short-to-moderate data lengths where highly accurate MA estimates are (essentially) impossible to obtain by any means. In summary, all of the ingredients have been put together for robust ARMA parameter estimation, as described in [9].

Two subsidiary topics were also the subject of some research. First, to visually demonstrate how parameter estimation accuracy depends on AR parameter values, the study reported in [7] was carried out. In terms of a mean square prediction error distortion measure, performance contours were obtained numerically for second order models. The results agree with intuition obtained gained from a knowledge of the Cramer-Rao bound on parameter estimation accuracy, namely that models with poles far apart and close to the unit circle correspond to models where high accuracy can be obtained. Second, as the result of research on robust state estimation for linear systems (i.e. robust Kalman filtering), a new linear filtering formulation in the square-root information filter family was discovered. The method has computational efficiencies in certain applications, especially where singular state transition matrices are encountered. Work on the development of this new filter was recently completed and will soon appear [8].

The final research topic on time series modeling is concerned with modeling of signals with abruptly changing statistical characteristics. The approach adopted, as

reported in [10], was to model such a signal as piecewise-autoregressive. Adaptive lattice algorithms for time-varying AR models have been widely studied in the signal processing literature. The new feature included in our approach to this problem is to add a second level of adaptation by switching the "forgetting factor" in the adaptive algorithm between large and small values according to whether or not a change in statistics has been detected. Various detection statistics have been subjected to experimental (and some analytical) evaluation, those chosen are all readily computed from signals that arise in the gradient adaptive lattice algorithm. The switching between large and small adaptation gains allows close, highly accurate tracking of parameters during periods of stationarity and fast response to abrupt changes that are detected

Our research also involved a second general topic, namely the formulation of signal modeling (parameter estimation) problems in a system-theoretic context. We begin our description with some notation. Let $(U_1, U_2, \cdots)$ be a sequence of observed random variables whose probability distributions are described by a parametrized family of density functions $\{p_k(u_1, \cdots, u_k; \theta)\}$. We assume that there exists a sequence of sufficient statistics for θ, $(T_1(U_1), T_2(U_1, U_2), \cdots)$, so that all of the information about the parameter θ available in the observations up to instant k is preserved in the (fixed dimension!) statistic T_k. This formulation is applicable to many parameter estimation problems arising in applications, including linear predictive (i.e. autoregressive) modeling of signals

In the study of problems where the observation record increases, it is a natural system theoretic view to regard a sequence of sufficient statistics as defining the input/output map of a dynamical system. In work done in collaboration with Eduardo Sontag of Rutgers University, we have developed some realization theory for nonlinear, time varying systems that can be applied to the study of state space models for sufficient sequences [2]. The importance of state space models should be clear: they provide a recursive means of updating the sequence of sufficient statistics as more observations are made. Actually, for this to be the case, it is necessary that the state space models be finite dimensional. Results in [2] indicate classes of systems for which finite dimensional realizations may be obtained and give a criterion for verifying that a (nonlinear, time

varying) state space model is a minimum dimension realization of its input/output map. The latter is an appropriately formulated theorem of the usual kind: controllability and observability of a realization imply that the realization is minimum dimension.

In [2] we adopted the framework of nonlinear filtering theory, i.e. the Bayesian perspective, and proved that $3p+1$ is the minimum dimension realization of the recursive Bayes estimator for the parameters of a p^{th} order autoregressive process. This estimator is based on reproducing (or conjugate) families of densities, but it is equivalent, at least for long observation records, to the usual least squares estimates used in linear predictive modeling.

Another subject considered in [2] is the relation of the notion of realizability of a sufficient sequence with other properties introduced in the statistics literature, especially transitivity. We showed that realizability has some important practical implications through its connection with (finite dimensional) recursive equations for updating the sequence of statistics that are not necessarily associated with transitivity.

We followed up our work on realizability with some further study of its implications and limitations. The theory in [2] requires that realizations take the form

$$X_{k+1} = \sigma_k(X_k, U_k)$$

$$T_k = \eta_k(X_k, U_k)$$

where the functions σ and η are taken to be differentiable, since we need some kind of smoothness condition to make sense out of the notion of dimension. In [2] we gave the following example to show that there are sufficient sequences admitting no smooth finite dimensional realization. For observations from a stationary Gaussian process with unknown mean and known, nonrational power spectral density function, there is a one-dimensional sufficient sequence defining a linear input/output map that has no smooth finite dimensional realization, linear or nonlinear. Viewed from the perspective of the observed process, this result is perhaps not unexpected, since there is no smooth finite dimensional model which generates such observations.

In following up on this example, we investigated its essential feature, namely that there is no finite dimensional model for the observation process, and we showed that this is the underlying feature of any example of this sort. Specifically, a realizability assumption, in conjunction with existence of a sufficient sequence, implies that the observation process is equivalent to one that is generated by a finite dimensional dynamical system driven by a sequence of independent random variables. (Here equivalence is in the natural sense for stochastic processes, namely the same family of distribution functions.) These results are described in [3], along with a discussion of how parametrized families of systems play a more natural role for the notion of "finitely generated" stochastic process than the approach involving sufficient sequences.

In further work, reported in summary form in [5], we have extended our approach to deal with the problem of prediction (where the parameters of the joint probability density function of the observations are of no interest). In practice, prediction problems usually arise in connection with a growing observation record instead of a fixed one. As described above, if there exists a sequence of sufficient statistics for the parameter, as in the case of an autoregressive process, then the sequence may be viewed as the input-output map of a dynamical system and the question of finite dimensional realizability may be investigated. For prediction problems, there are interesting relationships between realizability and a property called total sufficiency. For autoregressive processes, this property holds and provides one way of obtaining parameter-independent predictions. Unfortunately, it appears to be a very difficult computational task, being highly nonlinear, to obtain "optimum" predictors based on sufficient statistics. Whether this approach can be transformed into a useful one by reformulating it with a better parametrization, e.g. the lattice filter, remains as a challenging problem for further research. We remain convinced of its importance as a basic part of the understanding of how to do robust prediction.

Publications supported through grant AFOSR-84-0381

A list of publications prepared with the support of the grant follows, numbered according to the citations in the preceding report of accomplishments. In addition, the work described in the Ph.D. thesis of Dr. Shiping Li, now an Assistant Professor in the Department of Electrical Engineering at Texas A&M University, was supported by the grant. The thesis, entitle *Applications of Lattice Filters in Estimation of Times Series Models*, was completed in August, 1986. Essentially all of the work in the thesis is reported in publications listed below, but for completeness a copy of the Abstract of Dr. Li's dissertation is attached to this report.

1. S. Li and B.W. Dickinson, "Robust estimation of AR models using the lattice structure," *Proceedings, 22nd Allerton Conf. on Computing, Communication, and Control*, Univ. of Illinois, 1984, pp. 299-306.

2. B.W. Dickinson and E.D. Sontag, "Dynamical realizations of sufficient sequences," *IEEE Trans. on Information Theory*, vol. IT-31, 1985, pp. 670-676; also *Abstracts of Papers, International Symposium on Information Theory*, Brighton, England, 1985, p 132

3. B.W. Dickinson, "Sufficient sequences and state space models for random processes," *Proc. 24th IEEE Conf. on Decision and Control*, 1985, pp. 396-398; also *IEEE Trans. on Automatic Control*, vol. AC-31, 1986, pp. 89-91.

4. S. Li and B.W. Dickinson, "An efficient method to compute consistent estimates of the AR parameters of an ARMA model," *Proc. 24th IEEE Conf. on Decision and Control*, 1985, pp. 1072-1076; also *IEEE Trans. on Automatic Control*, vol. AC-31, 1986, pp. 275-278.

5. B.W. Dickinson, "Sufficient statistics and prediction of autoregressive time series," (Summary only) *Proc. 19th Conf. on Information Sciences and Systems*, The Johns Hopkins University, 1985, p. 236.

6. S. Li and B. W. Dickinson, "Application of lattice filters to robust estimation of AR and ARMA models," (Abstract only) *Abstracts of Papers, International Symposium*

on Information Theory. Ann Arbor, MI, 1986, p. 52.

7. S. Li and B. W. Dickinson, "Performance contours of autoregressive estimates," to appear in *IEEE Trans. Acoust., Speech, Signal Process.*, 1987.

8. C. G. Boncelet, Jr. and B. W. Dickinson, "An Extension to the SRIF Kalman Filter," to appear in *IEEE Trans. Automat. Contr.*, 1987.

9. S. Li and B. W. Dickinson, "Application of the lattice filter to robust estimation of AR and ARMA models," submitted to *IEEE Trans. Acoust., Speech, Signal Process.*

10. S. Li and B. W. Dickinson, "Jump detection and fast parameter tracking for piecewise AR processes using adaptive lattice filters," to be presented at and to appear in the proceedings of *1987 Intl. Conf. on Acoustics, Speech, and Signal Processing*, Dallas, TX, April 1987

Applications of Lattice Filters in Estimation of Time Series Models

Shiping Li

Abstract

This dissertation is concerned with estimation problems involving ARMA models and some generalizations of AR models using lattice filter algorithms. The generalizations of AR models considered are AR processes in additive noise and piecewise AR processes.

After an introduction, in Chapter 2 we present a new method to estimate the parameters of Gaussian stationary ARMA models based on an iterated least squares regression approach. It is shown that a $(p+q)$-stage lattice whitening filter can be used to obtain consistent estimates of the AR parameters of an ARMA(p, q) model. It is also shown that a set of MA parameter estimates can be obtained using this approach with little added computation.

In Chapter 3, we consider the problem of estimating the AR parameters of an ARMA process in independent additive noise of two types. For white Gaussian additive noise, it is shown that the results obtained in Chapter 2 can be used to estimate the AR parameters consistently; for additive outliers, robust estimates of the AR parameters are obtained using a data cleaning operation to remove the contamination.

In Chapter 4, we define a new model, a *piecewise* AR process, for a class of time series whose the statistical properties may change abruptly ("jump") at some

unknown time points. For such a model we consider the problems of jump detection and fast tracking of the changing parameters. A method based on the adaptive least squares lattice filter algorithm is proposed to solve the problems. The method automatically detects the occurrences of jumps and adjusts the adaptation rate of the least squares adaptive lattice algorithm, so both accurate estimates and fast tracking ability is achieved.

In Chapter 5, we consider the problem of joint prediction and parameter estimation for Gaussian AR processes using the MAP criterion. It is shown that the estimates obtained can be expressed in terms of a set sufficient statistics for the parameters and prediction. However, exact MAP estimation is a difficult nonlinear problem. Some simulation results are presented to provide a comparison of approximate MAP estimates with estimates derived from conventional least squares methods.

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